Find your optimal f

by Ralph Vince

As a computer programmer for Larry Williams, winner of the 1987 World Cup Championship of Futures Trading, I have had the opportunity to analyze many good trading systems. Even the best systems, however, suffer from an as-yet incomplete library of money management techniques. I felt a technique to select the optimal number of contracts to trade in an account needed to be designed. The value of such a system is determined by using the formula for mathematical expectation:

$$(W \times P)+[L\ (1 - P)]=ME$$

Where:

P = The probability of winning

W = The amount you may win

L = The amount you may lose

ME = Mathematical expectation

Suppose someone offers to pay you $2 if a coin toss comes up heads, but you must pay $1 if the result is tails. The mathematical expectation here is a positive 50 cents, because

$$(2 \times 0.5) + [-1\ (1 - 0.5)] = 0.50$$

The mathematical expectation tells us what we can expect to make (or lose) on average with each bet. If the mathematical expectation is greater than zero, you have an edge. If you bet 1,000 times on the above example you can expect to be ahead by $500 (0.50 × 1,000) at the end of 1,000 tosses. This is predicated on betting $1 on each and every toss.

It is very unlikely, however, you would bet this way. As your winnings amassed, you would probably step up your wager to some fraction of your total stake. As soon as you do, you lower your mathematical expectation.

A winning situation can be turned into a losing one simply by not knowing the right amount to bet.

Suppose you have $1 to bet where your odds are 2-to-1 and you bet that $1 on the first toss and win. Now you have $3 in your stake, and again you bet your entire stake on the next toss and lose. At this point you are tapped out. Had you bet $1 on each toss you would now be ahead $1 . A winning situation can be turned into a losing one simply by not knowing the right amount to bet.

Once you arrive at the correct percentage, however, your stake will grow exponentially faster than with any other strategy.

Figure 1 represents our 2-to- 1 coin toss gain after 40 tosses —20 sequences of winning 2 then losing 1.

The optimal fraction (f value) to bet is 0.25. But look what happens if you are only off by 15%. At 0.1 and 0.4 the system doesn't make half as much. At 0.5 on up you are losing money!

How do we find this optimal fraction? Those familiar with John L. Kelly will immediately respond, "Just use Kelly." (You may have read John L. Kelly's 1956 article in the *Bell System Technical Journal*. Many have used its formulas to pick the best gambling fractions.) This is not correct, because Kelly's method will only give you the optimal fixed fraction when all wins are the same amount and all losses are the same amount. Our 2-to- 1 coin toss example is such a case.

When you win and lose different amounts, as in trading, an entirely different technique is needed to find the optimal fixed fraction (f). Such a procedure requires you to produce a list of profits and losses (P&L) that a trading system has experienced for a given market. In the following example assume four trades with P&Ls of -1, -3, 3, 5.

Set up a table with a column for P&Ls, a cumulative column and a column you have converted the cumulative column from dollar amounts to holding period returns (HPR) (Figure 2).

The cumulative column is not the same as a cumulative listing for all the trade profits and losses. To calculate your cumulative entry for each trade you must take the previous entry and divide it by the critical value. (For the first trade this answer will be 1 because the previous cumulative entry is the same as the critical value.) Multiply the answer by the current trade amount. Finally, add this answer and the previous cumulative entry to obtain the current cumulative entry. HPRs are found by dividing each cumulative entry by the previous cumulative entry.

You need a starting value to begin the division process. This starting value is the critical value. First, take the largest loss and divide it by 0.01. Now make it a positive value. The critical value for our example sequence is:

3 / 0.01 = 300

Once you have the HPRs for all the trades calculated, you can then find the Terminal Wealth Relative (TWR) of a given series. The TWR is simply the product of all HPRs multiplied together. This is the factor you would have made on your money had you been trading the fixed fraction (Figure 2). The TWR will always be the same, regardless of the sequence of trades!

Now run through the entire procedure again, this time increasing your critical value. Instead of dividing the largest loss by 0.01, divide it by 0.02. Since 3 /0.02 = 150, you will start this next sequence at 150 (Figure 3). The HPRs are simply the cumulative column divided by the previous cumulative. The TWR is the product of all HPRs multiplied together. Repeat the procedure, this time dividing the largest loss by 0.03, then 0.04 and so forth. Continue until you divide the largest loss by 1.00. You are testing different f values.

If you plot the results on an x-y plane, where the y-axis is the TWRs and the x-axis the respective f values, you will obtain a function that is bell-shaped, similar to the one shown in Figure 1. The optimal f for a given market system is that value for f showing the highest TWR.

Once you have your optimal f (which in our trade example is 0.32), you then divide your equity by the critical value corresponding to that f, and that's how many contracts to trade. If the optimal f is 0.32 and the biggest loss is -3, then the critical value is 3 /0.32 = 9.375.

Therefore, you would trade one contract for every $9.375 in your stake. If you had $20 in your stake, you would trade two contracts. Once your stake grew to $28.125 you would start trading three contracts. If your stake dipped to $ 18.74 or less, you would trade only one contract. Of course, these values are small for expositional purposes!

Ralph Vince created portfolio model techniques to combine different market systems using optimal f. "Can you find your optimal f?" originally appeared in the June 1989 Future Markets Alert, published by the Robbins Trading Co.

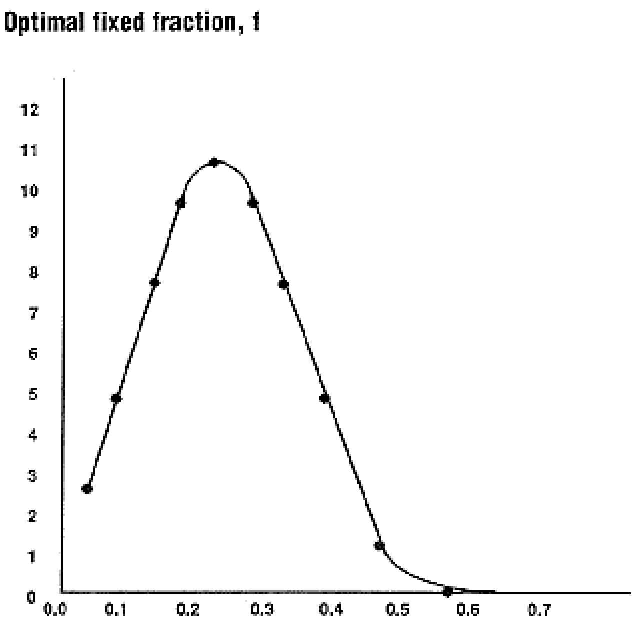

FIGURE 1: *This figure represents our 2 - to - 1 coin toss gain after 40 tosses - 20 sequences of winning 2 then losing 1. The optimal fraction to bet is 0.25. But look what happens if you are only off by 15%. At 0.5 on up you are losing money!*

Holding period returns		
P&L	**CUM**	**HPR**
	300	
–1	299	0.9966
–3	296	0.99
3	299	1.0101
5	304	1.016
TWR		1.0132

The TWR is simply the product of all HPRs multiplied together. This is the factor you would have made on your money had you been trading the fixed fraction. The TWR will always be the same, regardless of the sequence of trades.

FIGURE 2: *The TWR is simply the product of all HPRs multiplied together. This is the factor you would have made on your money had you been trading the fixed fraction. The TWR will always be the same, regardless of the sequence of trades.*

Holding period returns		
P&L	**CUM**	**HPR**
	150	
–1	149	0.9933
–3	146	0.9799
3	149	1.0205
5	154	1.0336
TWR		1.0267

Increase your critical value this time around and see what happens.

FIGURE 3: *Increase your critical value this time around and see what happens.*